Chapter 6: Equations

Equation 6.1:

$$d \sim \mathrm{N}\left(\delta, \mathrm{V}\left(d\right)\right)$$

Equation 6.2:

$$d^* = \frac{d - \delta}{+\sqrt{\mathrm{V}\left(d\right)}}$$

$$\sqrt{}$$

Equation 6.3:

$$\mathrm{E}\left(d^*\right) = \frac{1}{+\sqrt{\mathrm{V}\left(d\right)}}\left(\mathrm{E}\left(d\right) - \mathrm{E}\left(\delta\right)\right) = \frac{1}{+\sqrt{\mathrm{V}\left(d\right)}}\left(\delta - \delta\right) = 0$$

Equation 6.4:

$$\mathrm{V}\left(d^*\right) = \left(\frac{1}{\mathrm{V}\left(d\right)}\right)\mathrm{V}\left(d\right) = 1$$

Equation 6.5:

$$d^* = \frac{d - \delta}{\mathrm{SD}\left(d\right)} \sim t^{(df)}$$

Equation 6.6:

$$1 - \alpha = \mathrm{P}\left(-t_{\alpha/2}^{(df)} < d^* < t_{\alpha/2}^{(df)}\right)$$

Equation 6.7:

$$1-\alpha = \mathrm{P}\left(-t_{\alpha/2}^{(df)} < \frac{d-\delta}{\mathrm{SD}(d)} < t_{\alpha/2}^{(df)}\right)$$

Equation 6.8:

$$1-\alpha = \mathrm{P}\left(d - t_{\alpha/2}^{(df)}\mathrm{SD}(d) < \delta < d + t_{\alpha/2}^{(df)}\mathrm{SD}(d)\right)$$

Equation 6.9:

$$\mathrm{SD}(\overline{y}) = \frac{\mathrm{SD}(y_i)}{\sqrt{n}}$$

Equation 6.10:

$$.95 = \mathrm{P}\left(\overline{y} - t_{.025}^{(n-1)}\frac{\mathrm{SD}(y_i)}{\sqrt{n}} < \mu < \overline{y} + t_{.025}^{(n-1)}\frac{\mathrm{SD}(y_i)}{\sqrt{n}}\right)$$

Equation 6.11:

$$\begin{aligned} .95 &= \mathrm{P}\left(28,415 - 2.093\frac{30,507}{\sqrt{20}} < \mu < 28,415 + 2.093\frac{30,507}{\sqrt{20}}\right) \\ &= \mathrm{P}(14,137 < \mu < 42,693) \end{aligned}$$

Equation 6.12:

$$\begin{aligned} .95 &= \mathrm{P}\left(29,146 - 1.960\frac{42,698}{\sqrt{1,000}} < \mu < 29,146 + 1.960\frac{42,698}{\sqrt{1,000}}\right) \\ &= \mathrm{P}(26,500 < \mu < 31,792) \end{aligned}$$

Equation 6.13:

$$\delta_0 - t_{\alpha/2}^{(df)}\mathrm{SD}(d) < d < \delta_0 + t_{\alpha/2}^{(df)}\mathrm{SD}(d)$$

Equation 6.14:

$$1-\alpha = \mathrm{P}\left(\delta_0 - t_{\alpha/2}^{(df)}\mathrm{SD}(d) < d < \delta_0 + t_{\alpha/2}^{(df)}\mathrm{SD}(d)\right)$$

Equation 6.15:

$$.95 = \mathrm{P}\left(\mu_0 - t_{.025}^{(n-1)}\frac{\mathrm{SD}(y_i)}{\sqrt{n}} < \overline{y} < \mu_0 - t_{.025}^{(n-1)}\frac{\mathrm{SD}(y_i)}{\sqrt{n}}\right)$$

Equation 6.16:

$$\begin{aligned} .95 &= \mathrm{P}\left(25{,}000 - 2.093\frac{30{,}507}{\sqrt{20}} < \overline{y} < 25{,}000 - 2.093\frac{30{,}507}{\sqrt{20}}\right) \\ &= \mathrm{P}(10{,}722 < \overline{y} < 39{,}278) \end{aligned}$$

Equation 6.17:

$$\begin{aligned} .95 &= \mathrm{P}\left(25{,}000 - 1.960\frac{42{,}698}{\sqrt{1{,}000}} < \overline{y} < 25{,}000 + 1.960\frac{42{,}698}{\sqrt{1{,}000}}\right) \\ &= \mathrm{P}(22{,}354 < \overline{y} < 27{,}646) \end{aligned}$$

Equation 6.18:

$$1-\alpha = \mathrm{P}\left(-t_{\alpha/2}^{(df)} < \frac{d-\delta_0}{\mathrm{SD}(d)} < t_{\alpha/2}^{(df)}\right)$$

Equation 6.19:

$$d_0^* = \frac{d - \delta_0}{\mathrm{SD}(d)}$$

Equation 6.20:

$$1 - \alpha = \mathrm{P}\left(-t_{\alpha/2}^{(df)} < \frac{d - \delta_0}{\mathrm{SD}(d)} < t_{\alpha/2}^{(df)}\right) = \mathrm{P}\left(-t_{\alpha/2}^{(df)} < d_0^* < t_{\alpha/2}^{(df)}\right)$$

Equation 6.21:

$$t_{\alpha/2}^{(df)} \le \left|d_0^*\right|$$

Equation 6.22:

$$\mu_0^* = \frac{\bar{y} - \mu_0}{\mathrm{SD}(\bar{y})}$$

Equation 6.23:

$$\mu_0^* = \frac{\bar{y} - \mu_0}{\mathrm{SD}(\bar{y})} = \frac{28,415 - 25,000}{30.507 / \sqrt{20}} = .501$$

Equation 6.24:

$$\mu_0^* = \frac{\bar{y} - \mu_0}{\mathrm{SD}(\bar{y})} = \frac{29,146 - 25,000}{42,698 / \sqrt{1,000}} = 3.071$$

Equation 6.25:

$$p\text{-value} > \alpha$$

Equation 6.26:

$$p\text{-value} \leq \alpha$$

Equation 6.27:

$$\mathrm{P}\left(t^{(19)} \geq .501\right) > .25$$

Equation 6.28:

$$\mathrm{P}\left(t^{(\infty)} \geq 2.576\right) = .005$$

Equation 6.29:

$$1 - \alpha = \mathrm{P}\left(\frac{d - \delta}{\mathrm{SD}(\delta)} < t_{\alpha}^{(df)}\right)$$

Equation 6.30:

$$1 - \alpha = \mathrm{P}\left(d < \delta_0 + t_{\alpha}^{(df)}\mathrm{SD}(d)\right)$$

Equation 6.31:

$$H_1 : \mu_1 > 25,000$$

Equation 6.32:

$$.95 = \mathrm{P}\left(\bar{y} < \mu_0 + t_\alpha^{(df)}\mathrm{SD}\left(\bar{y}\right)\right)$$

Equation 6.33:

$$.95 = \mathrm{P}\left(\bar{y} < 25,000 + 1.729\frac{30,507}{\sqrt{20}}\right) = \mathrm{P}\left(\bar{y} < 36,794\right)$$

Equation 6.34:

$$\mathrm{P}\left(\text{Type II error}\right) = \mathrm{P}\left(d \le \delta_0 + t_\alpha^{(df)}\mathrm{SD}\left(d\right) \mid \delta = \delta_1\right)$$

Equation 6.35:

$$d_1^* = \frac{d - \delta_1}{\mathrm{SD}\left(d\right)} \sim t^{(df)}$$

Equation 6.36:

$$\begin{aligned}\mathrm{P}\left(\text{Type II error}\right) &= \mathrm{P}\left(\frac{d - \delta_1}{\mathrm{SD}\left(d\right)} \le \frac{\delta_0 + t_\alpha^{(df)}\mathrm{SD}\left(d\right) - \delta_1}{\mathrm{SD}\left(d\right)}\right) \\ &= \mathrm{P}\left(t^{(df)} \le \frac{\delta_0 + t_\alpha^{(df)}\mathrm{SD}\left(d\right) - \delta_1}{\mathrm{SD}\left(d\right)}\right)\end{aligned}$$

Equation 6.37:

$$\text{power} = 1 - \mathrm{P}\left(\text{Type II error}\right)$$

Equation 6.38:

$$\mathrm{P}\left(\text{Type II error}\right)=\mathrm{P}\left(t^{(df)}\leq\frac{\mu_0+t_{\alpha}^{(df)}\mathrm{SD}\left(\bar{y}\right)-\mu_1}{\mathrm{SD}\left(\bar{y}\right)}\right)$$

Equation 6.39:

$$\mathrm{P}\left(t^{(19)}\leq .688\right)>\mathrm{P}\left(t^{(19)}\leq .263\right)>\mathrm{P}\left(t^{(19)}\leq 0\right)$$

Equation 6.40:

$$\mathrm{P}\left(\text{Type II error}\right)=\mathrm{P}\left(\frac{d-\delta_1}{\mathrm{SD}\left(d\right)}\leq\frac{-\left(\delta_1-\delta_0\right)+t_{\alpha}^{(df)}\mathrm{SD}\left(d\right)}{\mathrm{SD}\left(d\right)}\right)$$

Equation 6.41:

$$\frac{-\left(\delta_1-\delta_0\right)}{\mathrm{SD}\left(d\right)}+t_{\alpha}^{(df)}$$

Equation 6.42:

$$1-\alpha=\mathrm{P}\left(d-2t_{\alpha/2}^{(df)}\mathrm{SD}\left(d\right)\leq d\leq d\right)$$